# Direct numerical simulation of particle-fluid flows in turbulent mixing layer\*

ZHENG Youqu, FAN Jianren\*\*, ZHA Xudong and CEN Kefa

(Department of Energy Engineering, Zhejiang University, Hangzhou 310027, China)

Received May 28, 2001; revised June 28, 2001

Abstract The coherent structures of a three-dimensional temporally mixing layer and the associated dispersion patterns of particles are numerically studied using a pseudospectral method for fluid and the Lagrangian approach for tracing particles at different Stokes numbers without consideration of particle-particle interactions. The results show that the particles with Stokes number of the order of unity have the largest concentration near the outer edges of the large-scale spanwise vortex structures. The study validates the effect of the streamwise large-scale structures on the particle distribution along the spanwise and transverse directions and it enhances with the development of the three-dimensionality of the mixing layer, which results in a 'mushroom' shape of the particle distribution in the spanwise direction.

Keywords: direct numerical simulation, particle dispersion, mixing layer, pseudospectral method, Lagrangian approach

The plane mixing layer, as a classical field for the study of turbulence in free shear layers, was extensively studied both numerically and experimentally in the past decades. Understanding the fundamental of particle dispersion in turbulent shear flows is more important in many producing processes such as boiler combustion, particle feed jets and oil droplet-fueled gas combustors. Simulation of the three-dimensional evolution of a plane mixing layer was carried out recently in detail, in which the mechanisms responsible for the growth of three-dimensionality and onset of transition to turbulence were described<sup>[1,2]</sup>. Lasheras and Choi<sup>[3]</sup> visualized and characterized experimentally the development of the complex interactions between the small structures and the large ones.

The prediction of particle dispersion and the associated effects of the large-scale structures have also been extensively studied by numerical and experimental techniques. Crowe et al. [4] proposed the use of Stokes number of particles and concluded that the particles with small Stokes number might closely follow the turbulent flow and the particles conversely with large Stokes number should be little affected by the fluid flow. For intermediate Stokes number, however, particles might be dispersed significantly faster than the fluid motion due to the centrifugal ef-

fect created by the organized vortex structures. Recent efforts to predict three-dimensional features of particle dispersion were made by Marcu and Meiburg<sup>[5]</sup> for a plane mixing layer. The simulated results showed that the presence of the streamwise vortices would result in additional effects that modify the dispersion patterns of particles. Intense three-dimensional vortex stretching and folding could a produce 'mushroom' shape of particle distribution.

Owing to the improvement of computer techniques, direct numerical simulation (DNS) is becoming a very powerful tool to obtain three-dimensional, time-dependent solutions to the nonlinear Navier-Stokes equations. In this paper, we will firstly show the DNS results of the process of vortex rollup, pairing and development in flow field, and then analyze three-dimensional dispersion patterns of particle with different Stokes numbers. It is of significance to understand the role of coherent vortex structures on particle dispersion.

#### 1 Mathematical model

## 1.1 Flow-field simulation

The plane mixing layer consists of two parallel streams with different velocities  $U_1$  and  $U_2$  ( $U_1 > U_2$ ). The non-dimensional continuity and momentum equations for an incompressible flow without

<sup>\*</sup> Supported by the Major State Basic Research Development Program of China (Grant No. G 19990222)

<sup>\*\*</sup> To whom correspondence should be addressed. E-mail; fanjr@mail.hz.zj.cn

body force are:

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{F} - \nabla \Pi + \frac{1}{Re} \nabla^2 \mathbf{U}, \qquad (1)$$

$$\nabla \cdot \boldsymbol{U} = 0, \tag{2}$$

where  ${\bf F}={\bf U}\times {\bf \omega}$ ,  ${\bf \Pi}$  is the total pressure, vorticity  ${\bf \omega}=\nabla\times {\bf U}$ ,  ${\bf R}{\bf e}=U_0\theta_0/\nu$ , in which  $U_0=(U_1-U_2)$  and  $\theta_0$  (the initial momentum thickness) are characteristic velocity and length respectively.

The detailed initial conditions of flow field and the solve of governing equations could be found in our previous paper<sup>[6]</sup>.

### 1.2 Particle dispersion simulation

In the particle dispersion simulation, all particles are supposed to be rigid spheres with identical diameter  $d_p$  and density  $\rho_p$ . The flow can be considered as dilute so particle-particle interactions are neglected. The non-dimensional motion equation for a particle which is just considered the drag force is:

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = f/St(\mathbf{U} - \mathbf{V}),\tag{3}$$

in which V is the velocity of the particle, U the velocity of the fluid at the position of the particle and Stokes number is defined as  $St = \frac{\rho_{\rm p} d_{\rm p}^2/18\mu}{\theta_0/U_0}$ , f is the modification factor for the Stokes drag coefficient which is described as  $f = 1 + 0.15Re_{\rm p}^{0.687}$ ,  $R_{\rm ep} =$ 

 $|U - V| d_p/\nu$ . The velocity and position of particles can be obtained by integrating Eq. (3).

#### 2 Results and discussions

#### 2.1 DNS of coherent structures in the mixing layer

The two-dimensional contours of spanwise vorticity has been shown in Ref. [6].

Fig. 1 shows the three-dimensional contours of streamwise vorticity. The solid lines represent positive values of the vorticity and the dashed lines represent negative ones. The mechanism of streamwise vorticity origin and development are subject to threedimensional instability. It can be found that counterrotating streamwise rib vortices firstly occur in braids region then extend from the bottom of one roller to the top of the next, which is in good agreement with what experimentally observed by Lasheras and Choi<sup>[3]</sup>. Besides the ribs, the quadrupoles are the other notable vortices, which can be clearly observed in the figure (T = 35, dashed lines). The quadrupole occurs in the core region with the initial streamwise tube cutting and departing there because of stretching by the forming rollers. With the time increasing, two quadrupoles come together, and the rib between them is extruded and turns short (T = 65). Afterwards, the streamwise vortex tube stretch and gradually come up to approximately parallel vortex tube struc-

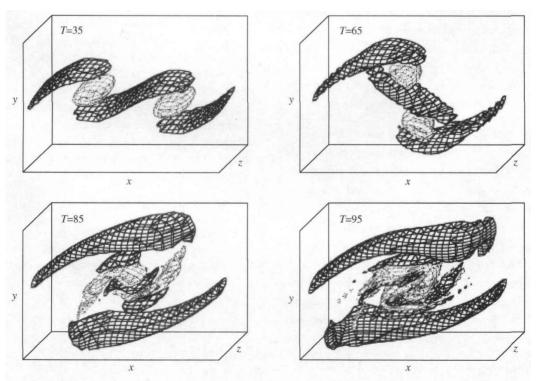


Fig. 1. Three-dimensional contours of streamwise vorticity

tures (T=85). Finally, small ribs are induced by larger ones which leads to vortices tubes' gradual amalgamation. In core regions, vortices are also regrouping, forming coherent pairing structures.

### 2.2 Particle dispersion by vortex structures

The particle dispersion with different Stokes numbers ranging from 0.01 to 100 is numerically simulated in this study. Initially, one particle is positioned at each computational cell and the velocity of each particle is set to the same as the fluid velocity at that position. To describe the overall concentration character of the particles,  $N_{\rm rms}$ , the root mean square of particle number per cell over the whole field, is used and defined as

$$N_{\rm rms} = \left(\sum_{i=1}^{n_{\rm t}} n_i^2 / n_{\rm t}\right)^{1/2},\tag{4}$$

where  $n_t$  is the total number of computational cells and  $n_i$  the number of particles in the *i*th cell. It is noted that  $n_t = 128 \times 128 \times 64$  and  $N_{\rm rms} = 1$  when the time T = 0.

Fig. 2. shows  $N_{\rm rms}$  values for particles with different Stokes numbers at different time. It is well known that the large-scale structures have important effects on the dispersion of the particles when the particle aerodynamic response time scale is of the same order of magnitude as the time scale associated with the large-scale organized structures. So it can be

found that the particles with Stokes number of order of unity apparently have larger  $N_{\rm rms}$  values, and the values of  $N_{\rm rms}$  increases with time.

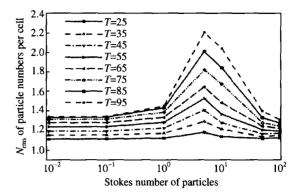


Fig. 2. N<sub>rms</sub> for particles with different Stokes number.

The different dispersion patterns for the particles with different Stokes numbers in  $Z=T_z/4$  plane at T=45 are shown in Fig. 3. The plane concept used here in the discussion of particle dispersion always corresponds to a thin slice with the thickness of a computational cell. For small values of Stokes number, the particles should closely follow the turbulent of the fluid and the particle or fluid dispersion ratio should be approximately unity. While for large values of Stokes number the particles should be less affected by the fluid motion and the particle or fluid dispersion ratio should approach zero. For the particles with Stokes number of the order of unity, the tendency to accumulate at the periphery of the larger-scale struc-

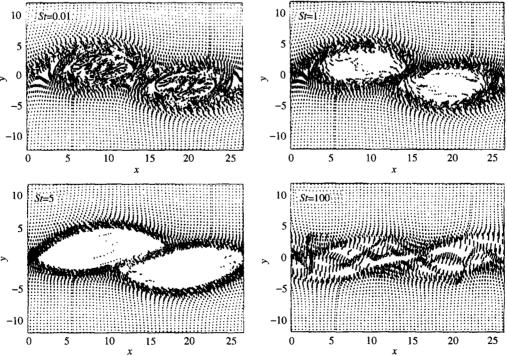


Fig. 3. Distribution of different particles in the  $Z = T_z/4$  plane at T = 45

tures is evident as a consequence of a centrifugal effect created by the organized nature of the large-scale structures, so the particle or fluid dispersion ratio could then potentially exceed unity. The different dispersion patterns for particles with various Stokes numbers can also be found in recent report of Wen et al. [7]

To study the particle dispersion levels quantitatively under the effect of the large-scale spanwise structures, the dispersion function in the transverse (Y) direction for particles distributed on the Y=0 plane is evaluated, which is obtained from

$$D_{y}(t) = \left(\sum_{t=1}^{n_{y}} (Y_{i}(t) - Y_{m}(t))^{2} / n_{y}\right)^{1/2}$$
 (5)

where  $n_y$  is the total number of particles distributed on Y=0 plane,  $Y_i(t)$  the particle displacement in the transverse direction at time t, and  $Y_m(t)$  the mean value. Fig. 4 shows the dispersion function of particles with different Stokes numbers. At small time interval, the particles at Stokes number of 0.01 show more dispersion in transverse direction. As time increases, the particles with Stokes number of the order of unity are more dispersed owing to the development of large-scale structures and the preferential effect of the structures on particle dispersion. For the particles with large Stokes numbers, the dispersion

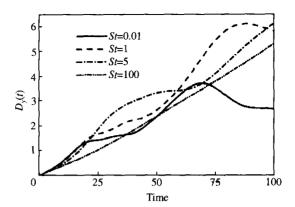


Fig. 4. Particle dispersion function with time in the transverse ( Y ) direction.

varies smoothly. These results are consistent with the result obtained by Chein and Chung<sup>[8]</sup>. It can be found from the figure that there are noticeable waves in the plot of the transverse dispersion function for particles with small Stokes numbers, which might be the effects of Kelvin-Helmholtz rollup and pairing.

To show how the streamwise large-scale structures affect the distribution of particles, mid-pairing plane ( $X = T_x/2$ ), a streamwise plane is observed. Fig. 5 gives the different distribution for particles with various Stokes numbers in this plane at T = 95. It is obvious that the particles with Stokes number of

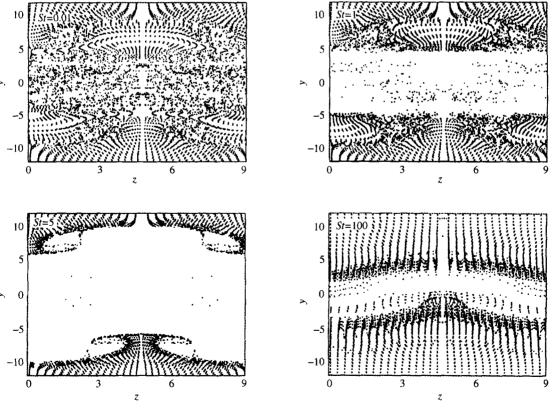


Fig. 5. Distribution of particles in the  $X = T_x/2$  plane (mid-pairing plane) at T = 95.

the order of unity concentrate most near the outer edges of large-scale spanwise vortex structures while other particles distribute more evenly in this plane. The variation of the concentration along the spanwise direction is subject to the streamwise vortex structures. For St=5, with the development of the streamwise structures, the variation of particle concentration along the spanwise direction increases and some particles 'burst' out from the high concentration area due to a pair of counter-rotating rib vorticies and finally develop a mushroom-shaped distribution.

#### 3 Conclusions

The direct numerical simulation results shown here demonstrate that the dispersion of particles in a plane mixing layer is closely connected with the large-scale organized structures. The particles with Stokes number of the order of unity tend to concentrate near the outer edges of the spanwise vortex structures. While the presence of the streamwise large-scale structures causes the variation of the particle distribution along the spanwise and the transverse directions, and the extent of variation also increases with the de-

velopment of the three-dimensionality, which results in the occurrence of a mushroom-shaped distribution of particles in spanwise direction.

#### References

- 1 Rogers, M. M. et al. The three-dimensional evolution in a plane of a plane mixing layer: the Kelvin-Helmholtz roll-up. Journal of Fluid Mechanics, 1992, 243: 183.
- 2 Moser, R. D. et al. The three-dimensional evolution in a plane of a plane mixing layer: paring and transition to turbulence. Journal of Fluid Mechanics, 1993, 247; 275.
- 3 Lasheras, J. C. et al. Three-dimensional instability of a plane free shear layer: an experimental study of the formation and evolution of streamwise vortices. Journal of Fluid Mechanics, 1988, 189: 53.
- 4 Crowe, C. T. et al. Particle dispersion by coherent structures in free shear flows. Particulate Science and Technology Journal, 1985, 3: 149.
- 5 Marcu, B. et al. Three-dimensional features of particle dispersion in a plane mixing layer, Physics Fluids. 1996, 8: 2266.
- 6 Zheng, Y. Q. et al. Direct numerical simulation of three-dimensional coherent structure in plane mixing layer. Progress in Natural Science, 2001, 11: 235.
- 7 Wen, F. et al. Particle dispersion by vortex structure in plane mixing layers. Journal of Fluids Engineering, 1992, 114: 657.
- 8 Chein, R. et al. Simulation of particle dispersion in a two-dimensional mixing layer. American Institute of Chemical Engineering Journal, 1988, 34: 946.